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ANODE-SHAPE DETERMINATION WITH ALLOWANCE FOR ELECTROLYTE PROPERTIES IN PROBLEMS OF DIMENSIONAL ELECTROCHEMICAL MACHINING OF METALS

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Based on the boundary-element technique, a method for determining the contour of an anode-workpiece with a prescribed shape of a cathode-tool is suggested for plane problems of dimensional electrochemical machining of metals. Within the scope of the assumptions made, the original problem is reduced to the problem of a fictitious flow of an ideal fluid with free surfaces. The allowance for the machining mode and electrolyte properties yields a nonlinear condition at the free surface.

Key words: *electrochemical machining of metals, shape of the anode boundary, boundary-element method.*

Introduction. Dimensional electrochemical machining (DECM) of metals is an advanced method for the production of workpieces from metals and alloys with a specified shape, size, and surface quality [1]. The method is based on the principle of the local dissolution of the anode — workpiece in the electrolyte flow. The cathode — machining tool — is represented by an electrode with a specified surface shape. The electrochemical dissolution rate V of the metal, according to Faraday's law, is given by the expression $V = \eta\varepsilon i$, where η is the current effervescive for the reaction of anode dissolution of the metal, i is the current density, and ε is the electrochemical equivalent of the metal. The current effervescive η allows for the processes that occur at the anode surface, which accompany metal dissolution, and is equal to the proportion of the charge spent on anode dissolution of the metal only. Electrochemical processes are caused by electrolyte-solution pumping through the interelectrode gap to remove reaction products (gas and sludge) as well as released heat from the machining area. To improve machining precision, the gap between the electrodes for DECM is made small (about 0.1 mm) by providing a specified cathode-feed velocity towards dissolution.

Two stages of machining could be distinguished: 1) workpiece machining in an unsteady regime (here, the law of the dissolution-rate distribution over the workpiece surface and local interelectrode gaps vary in time); 2) steady regime of machining where metal dissolution at all points of the workpiece surface occurs with a velocity distribution that ensures parallel motion of anode points with a velocity equal to the cathode-feed velocity.

Model of the Process. Electrochemical machining is a complicated process, which is described, in the general case, by a system of equations of a viscous multiphase electricity-conducting fluid, Maxwell equations, equations of energy, convection diffusion, gas state, and dependences of thermophysical parameters on temperature, pressure, and medium composition. The mathematical model of DECM reduces to determining the shape of one electrode with a given shape of the other. When both cathode shape and machining condition are specified, the problem is called direct. The inverse problem determines the shape of the cathode-tool that, under certain machining conditions, provides a workpiece of a required configuration [1].

Solving this problem is hindered by the fact that the workpiece boundary is unknown. Therefore, the ideal process model is used as the first approximation in the theoretical analysis of the DECM process. The main assumptions of the model and their detailed justification can be found in [1]. According to this model, in the case of a constant-current DECM, the electric field in the interelectrode gap can be considered as potential, i.e.,

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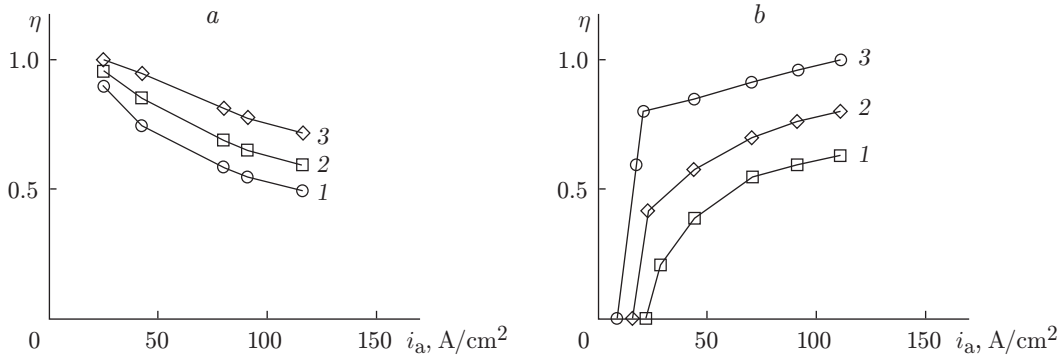

Fig. 1. Current effervescive versus the anode current density for machining 5KhNM steel in NaCl and $NaNO_3$ solutions with various concentrations: (a) NaCl with $C = 5$ (1), 10 (2), and 15% (3); (b) $NaNO_3$ with $C = 10$ (1), 15 (2), and 20% (3).

TABLE 1

NaCl		$NaNO_3$	
C, %	k	C, %	k
5	0.989	10	−1
10	0.972	15	−0.985
15	0.951	25	−0.983

$\boldsymbol{E} = -\operatorname{grad} u$ ($\boldsymbol{E}$ is the vector of electric-field strength and u is the electric-field potential). In an ideal DECM process, the electric field can be described by the Laplace equation $\nabla^2 u = 0$. The values of the potential u_a and u_c at the anode and cathode surfaces are constant [1].

Provided the necessary conditions are fulfilled, long-time machining causes a definite and time-constant change in the workpiece shape, which is called steady or stationary. In the steady mode, the workpiece-surface shape in the moving coordinate system attached to the cathode remains unchanged. This implies that the anode surface moves together with the cathode at a constant velocity V_c. In this case, the linear velocity of anode dissolution $V_a = \eta\varepsilon i/\rho$ in the direction normal to the anode surface at every point of the anode equals

$$V_a = V_c \cos\theta = (\boldsymbol{V}_c, \boldsymbol{n}_a). \tag{1}$$

Here θ is the angle between the vector of the cathode-feed velocity $\boldsymbol{V}_c$ and the unit vector of the outward normal to the anode $\boldsymbol{n}_a$. From condition (1) the steady distribution of the current density i at the stationary anode boundary can be determined by the equality

$$\eta(i_a) i_a = (\rho V_c/\varepsilon)\cos\theta, \tag{2}$$

where i_a is the anode current density and ρ is the density of the anode material. The current effervescive η depends on various parameters of the process, mainly, on the anionic composition and concentration of the electrolyte, chemical composition and density of the metal, and current density. Condition (2) allows for the fact that η is a function of i.

Figure 1 shows the current effervescive versus the anode current density for machining 5KhNM steel in NaCl and $NaNO_3$ solutions with various concentrations C, which were experimentally obtained in [2]. It follows from Fig. 1 that the dependence $\eta(i_a)$ for the above-mentioned electrolytes is described by the hyperbolic equation

$$\eta = a_0 + a_1/i_a. \tag{3}$$

Here, a_0 and a_1 are constant coefficients. For the experimental data in Fig. 1, the values of Pearson's correlation coefficient k are obtained (Table 1).

Substituting (3) into (2), we obtain

$$i_a = -a_1/a_0 + (\rho V_c/(a_0\varepsilon))\cos\theta. \tag{4}$$

In the model of the ideal process, the current density is determined by Ohm's law $\boldsymbol{i} = æ\boldsymbol{E} = -æ\operatorname{grad} u$, where $æ$ is the specific electrical conductivity of the medium. Then, the equality $i_a = -æ\,\partial u/\partial n_a$ holds at the anode surface.

TABLE 2

NaCl			$NaNO_3$		
C, %	a	b	C, %	a	b
5	−0.301	2.401	10	0.210	1.280
10	−0.205	1.865	15	0.141	1.104
15	−0.127	1.467	25	0.077	0.931

Let us pass to the dimensionless variables $\psi = (u - u_c)/(u_a - u_c)$ and $n = n_a/H$ (H is the characteristic size). Then, condition (4) takes the form

$$-æ\frac{u_a - u_c}{H}\frac{\partial\psi}{\partial n} = -\frac{a_1}{a_0} + \frac{\rho V_c}{a_0\varepsilon}\cos\theta. \tag{5}$$

Let us introduce the characteristic current density $i_0 = \rho V_c/\varepsilon$ into our consideration. According to (2), it corresponds to the anode current density for the case of parallel electrode boundaries in the stationary DECM mode with the current effervescive equal to unity and the cathode-feed direction perpendicular to these boundaries. The characteristic length is determined by the expression

$$H = æ(u_a - u_c)/i_0. \tag{6}$$

The distance H determines the interelectrode-gap width if the above-given conditions for the characteristic current density are satisfied. With allowance for (6), the steady condition (5) takes the form

$$\frac{\partial\psi}{\partial n} = \frac{a_1}{a_0 i_0} - \frac{1}{a_0}\cos\theta. \tag{7}$$

With allowance for condition (7) in the model of the ideal process, the DECM problem in a dimensionless form reduces to obtaining one of the unknown boundaries in the following problem. The function ψ corresponding to the electric-field potential satisfies the Laplace equation in the interelectrode gap

$$\nabla^2\psi = 0.$$

The following conditions are satisfied at the boundaries of the electrodes:

$$\psi_a = 1, \qquad \psi_c = 0.$$

The steady condition is fulfilled at the anode boundary:

$$\frac{\partial\psi}{\partial n} = -(a + b\cos\theta), \qquad a = -\frac{a_1}{a_0 i_0}, \qquad b = \frac{1}{a_0}.$$

The coefficients a and b are found from the experimental data presented in Fig. 1. The calculation results for $i_0 = 100$ A/cm^2 are listed in Table 2. In electrically insulated sectors, the condition of current impermeability is satisfied:

$$\frac{\partial\psi}{\partial n} = 0. \tag{8}$$

In formulating and solving DECM problems, the hydrodynamic analogy [3] of the electric field is employed, according to which the plane potential electric field is replaced by a fictitious flow of an ideal incompressible fluid. If we introduce the complex potential of the electrostatic field $W = \varphi + i\psi$, where ψ is the dimensionless potential of the electric field, we obtain $\partial\psi/\partial n = V$ along the line $\psi = \text{const}$ (in the case of the hydrodynamic interpretation of the DECM problems, $\boldsymbol{V}$ is the velocity vector of the fictitious flow of an ideal incompressible fluid). In standard hydrodynamic terminology, the problem of anode-shape determination is called a free-boundary problem.

Formulation of the Problem. We consider a plane problem of steady electrochemical machining by a cathode-tool consisting of rectangular segments and a semicircular cylindrical step of radius R (Fig. 2). The cathode moves in the negative y direction along the ordinate axis. The interelectrode gap is bounded by the anode boundary Γ_1 and cathode boundary Γ_3. In numerical calculations, the incoming and outgoing fictitious flows are cut at the right angle to the original direction of velocity at certain finite distances from the step. The cutting lines correspond to the segments of inflow Γ_4 and outflow Γ_2. The anode boundary Γ_1 is unknown, and its position is to be obtained in the course of solving the problem.

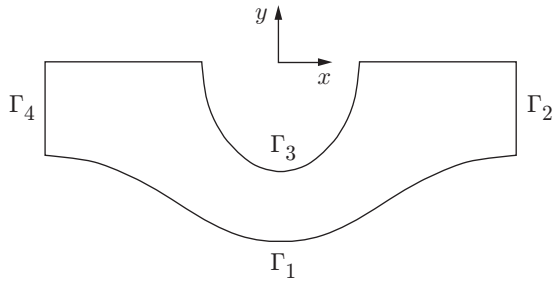

Fig. 2. Cathode-tool geometry.

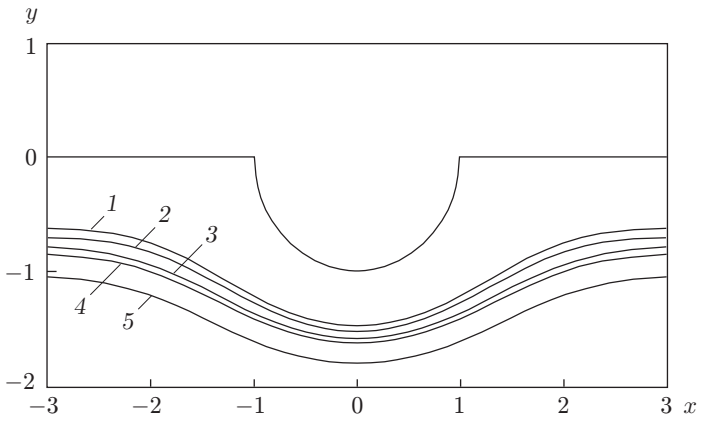

Fig. 3. Calculation results for anode boundaries: NaCl with $C = 10$ (1) and 15% (3); $NaNO_3$ with $C = 10$ (2), 15 (4), and 25% (5).

The problem reduces to solving the Laplace equation for the dimensionless potential ψ of the electrostatic field. At the domain boundaries, the function ψ satisfies the following conditions: $\partial\psi/\partial n = -(a + b\cos\theta)$ at the boundary Γ_1 (θ is the angle between the velocity vector of the fictitious flow and the abscissa axis), $\psi = 0$ at the boundary Γ_3, $\partial\psi/\partial n = 0$ on the cutting lines Γ_4 and Γ_2. The latter condition implies the absence of the velocity component normal to the main stream.

The problem is solved by the boundary-element method [4]. The anode boundary is found in accordance with iterative algorithms of free-surface construction considered in [5]. The method is based on the condition stipulating that the anode boundary is a streamline of the fictitious flow of an ideal fluid. In numerical calculations, the original position of the anode boundary is set arbitrarily. At all nodal points of the anode, we adopt the condition $\partial\psi/\partial n = -(a + b\cos\theta)$. The value of the potential found for each nodal point of the anode is compared with the value $\psi_a = 1$. The problem is considered to be solved if the difference between these two values is smaller than a specified error. Otherwise, the anode-boundary position securing the desired accuracy is iteratively selected.

Calculation Results. The problem is solved in dimensionless variables. The varied parameters of the problem are the step radius R and the parameters a and b. The interelectrode gap in the cross sections Γ_2 and Γ_4 is found by the formula

$$h = 1/(a + b). \tag{9}$$

Figure 3 shows the calculation results at $R = 1$ for 5KhNM steel in NaCl and $NaNO_3$ solutions of various concentrations (see Table 2).

Electrochemical Shaping by a Sectional Cathode. The use of sectional cathodes makes it possible to machine workpieces with large surfaces, such as, for instance, forging dies. We consider a plane problem of steady electrochemical machining by a two-section cathode-tool (Fig. 4). The cathode boundary consists of the rectangular sectors Γ_3 and Γ_5, which are boundaries of the sections, and the sector Γ_4 corresponds to the insulation boundary between the sections. The interelectrode gap is bounded by the anode boundary Γ_1, cathode boundary, and the inflow Γ_6 and outflow Γ_2 regions of the fictitious flow. The vector $\boldsymbol{V}_c$ shows the cathode-feed direction. The

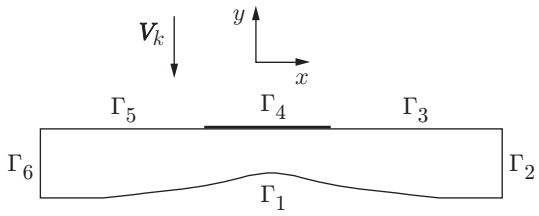

Fig. 4. Geometry of a two-section cathode-tool.

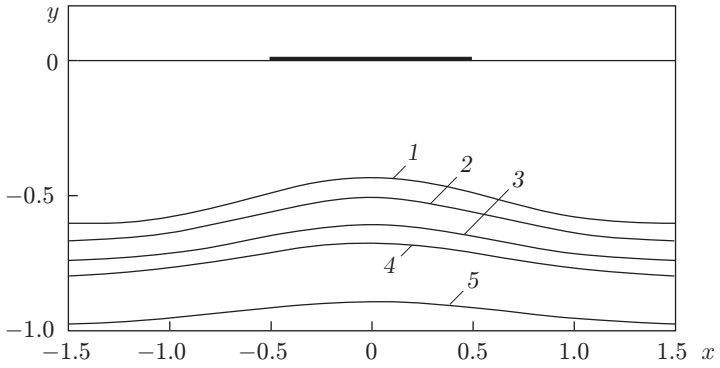

Fig. 5. Calculation results of anode boundaries for a two-section cathode-tool (notation the same as in Fig. 3).

function ψ satisfies the following conditions at the domain boundaries: $\partial\psi/\partial n = -(a + b\cos\theta)$ at the boundary Γ_1, $\psi = 0$ at the boundaries Γ_3 and Γ_5, and $\partial\psi/\partial n = 0$ on the cutting lines Γ_2 and Γ_6. In the sector Γ_4, condition (8) is satisfied.

The varied parameters of the problem are the dimensionless length L of the sector Γ_4 and the parameters a and b. The end-face gap in the cross sections Γ_6 and Γ_2 is calculated by formula (9). Figure 5 shows the calculation results at $L = 1$ for the solutions of NaCl and $NaNO_3$ with different concentrations (see Table 2).

Conclusions. The study implements a two-dimensional mathematical model of the ideal DECM process suggested in [1] with allowance for a particular dependence of the current effervescive on the anode current density. Steady-state shapes of the anode have been obtained for two configurations of the cathode-tool. The calculation results show that an increase in the electrolyte concentration (all other factors being equal) accelerates anode dissolution of the metal.

REFERENCES

1. A. D. Davydov and J. Kozak, *High-Velocity Electrochemical Shaping* [in Russian], Nauka, Moscow (1990).
2. F. V. Sedykin, B. P. Orlov, and V. F. Matasov, "Anode current effervescive of electrochemical machining applying constant and pulsed voltage," *Machine-Building Technology* (collected scientific papers) [in Russian], No. 39 (1975), pp. 3–7.
3. A. Kh. Karimov, V. V. Klokov, and E. I. Filatov, *Calculation Methods of Electrochemical Shaping* [in Russian], Izd. Kazan. Univ., Kazan' (1990).
4. C. A. Brebbia, J. Telles, and L. Wrobel, *Boundary Element Techniques*, Springer-Verlag, Heidelberg (1984)
5. K. E. Afanas'ev and S. V. Stukolov, "On the existence of three solutions for a supercritical steady-state flow of a heavy fluid over obstructions," *J. Appl. Mech. Tech. Phys.*, **40**, No. 1, 20–27 (1999).